

THE STRENGTH OF THE ANALOG AND GAMOW-TELLER GIANT RESONANCES AND HINDRANCE OF THE $2\nu\beta\beta$ - DECAY RATE

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Abstract

An approach for describing the hindrance of the nuclear $2\nu\beta\beta$ -decay amplitude is proposed. The approach is based on a new formula obtained by a model-independent transformation of the initial expression for the amplitude. This formula takes explicitly into account the hindrance of the decay-amplitude due to the presence of the collective Gamow-Teller state. Calculations are performed within the simplest version of the approach. Calculated and experimental $2\nu\beta\beta$ half-lives are compared for a wide range of nuclei.

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1. Introduction

A rough analysis of a great body of $2\nu\beta\beta$ -decay data shows that the nuclear decay-amplitude is hindered as compared to the one evaluated within the independent-quasiparticle approximation. (see e.g. refs [1,2]). The hindrance is caused by the presence of the isobaric analog and Gamow-Teller nuclear collective states (IAS and GTS) at a rather large excitation energy. These states exhaust a major fraction of, respectively, the Fermi and Gamow-Teller strength. To describe the hindrance, different versions of the quasiparticle random phase approximation (QRPA) are used in calculations of the nuclear $2\nu\beta\beta$ -decay amplitude. It has been found (see e.g. refs.[3]-[6]) that the calculated amplitudes are very sensitive to the particle-particle interaction strength. For this reason the predictive power of the current QRPA versions is poor. A similar instability of the calculated $0\nu\beta\beta$ - decay amplitude has also been found [7,8].

To bypass this difficulty, which probably is caused by an inconsistency in the current versions of QRPA, we propose a rather different approach to describe the hindrance of the nuclear $2\nu\beta\beta$ - decay amplitude. The method is based on a new formula obtained by a model-independent transformation of the initial expression for the amplitude. This formula takes explicitly into account the decay-amplitude hindrance due to the presence of the collective GTS. The approach is similar to the one widely used in the description of Fermi-type excitations in intermediate and heavy mass nuclei. These excitations can be described with the explicit use of the experimental fact, that the IAS exhausts most (more than 95%) of the Fermi sum rule $N - Z$. For these nuclei the mean nuclear Coulomb field is mainly responsible for the redistribution of the Fermi-strength from the IAS to its satellites. If the variable part of this field were neglected, the IAS would almost exhaust 100% of the Fermi strength and the Fermi $2\nu\beta\beta$ -amplitude would vanish for transitions to the ground and low-excited states of the product nucleus. This approach, which allows one to evaluate the small Fermi-strength of the IAS satellites populated virtually in the $2\nu\beta\beta$ -decay process, has been originally proposed by Lane [9] and is based on the approximate conservation of isospin symmetry in nuclei.

A similar situation occurs in the description of Gamow-Teller excitations. It is experimentally established that in intermediate and heavy mass nuclei the GTS exhausts about 70% - 80% of the sum rule $3e_q^2(N - Z)$, where $e_q \simeq 0.8$ is "an effective charge" describing, in particular, the re-normalization of the axial coupling constant for the weak interaction in nuclei. The GTS collectivity, directly related to this experimental fact, is explicitly used in the presented work to describe the hindrance of the nuclear $2\nu\beta\beta$ -decay amplitude. Early attempts along this line have been undertaken in refs. [10]-[12] within a perturbation theory. As a first step in applying the proposed approach, the decay amplitudes are calculated by the new formula using the BCS and the pair-vibration models to describe the corresponding states of isobaric nuclei.

Calculations performed for a wide range of nuclei show clearly the hindrance of the decay amplitude due to the presence of the collective GTS. The $2\nu\beta^-\beta^-$ and $2\nu\beta^+\beta^+$ (including the electron capture) half-lives are calculated using model parameters taken from independent data. Results of calculations are compared with available experimental data.

2. The basic formula for the nuclear $2\nu\beta\beta$ - decay amplitude

To make the following derivation of the new formula for the $2\nu\beta\beta$ -decay amplitude more transparent, we will use the same approach for the Fermi and the Gamow-Teller transitions. Let $G^{(\pm)} = \sum_a g_a^{(\pm)}$ be the operators of allowed Fermi ($g^{(\pm)} = \tau^{(\pm)}$) and GT ($g^{(\pm)} = \vec{\sigma}\tau^{(\pm)}$) β - transitions. First, we consider the $2\nu\beta^-\beta^-$ -decay amplitude. Let $|i\rangle$ and $|f\rangle$ be the ground state wave function, respectively, of a double-even parent nucleus $(Z, N + 2)$ and of a product nucleus $(Z + 2, N)$, E_i and E_f are the corresponding energies. The expression for the nuclear $2\nu\beta\beta$ - decay amplitude has the form (see e.g. refs. [13,14]):

$$M_G = \sum_S \langle f|G^{(-)}|S\rangle \langle S|G^{(-)}|i\rangle \omega_S^{-1} , \quad (1)$$

where the sum is taken over intermediate states of an isobaric nucleus $(N + 1, Z + 1)$ and $\omega_S = E_S - (E_i + E_f)/2$ is the excitation energy of an intermediate state. The $2\nu\beta^+\beta^+$ - decay amplitude is equal to conjugate value M_G^* .

We transform eq.(1) in the following way. Let $|G\rangle$ be one of the intermediate states with the energy E_G and $V_G^{(-)}$ be the following operator:

$$V_G^{(-)} = -i\hbar\dot{G}^{(-)} - \Delta_G G^{(-)}, \quad \Delta_G = E_G - E_i , \quad (2)$$

where a dot over the operator implies the time derivative. Matrix elements of this operator can readily be constructed:

$$\begin{aligned} -(\omega_G - \omega_S) \langle S|G^{(-)}|i\rangle &= \langle S|V_G^{(-)}|i\rangle , \\ -(\omega_G + \omega_S) \langle f|G^{(-)}|S\rangle &= \langle f|V_G^{(-)}|S\rangle , \end{aligned} \quad (3)$$

giving $\langle G|V_G^{(-)}|i\rangle = 0$. Using these matrix elements we transform amplitude (1) to the expression:

$$M_G = \sum_{S \neq G} \frac{\langle f|V_G^{(-)}|S\rangle \langle S|V_G^{(-)}|i\rangle}{\omega_S(\omega_G^2 - \omega_S^2)} - \frac{\langle f|V_G^{(-)}|G\rangle \langle G|G^{(-)}|i\rangle}{2\omega_G^2} . \quad (4)$$

The last term in this expression can be transformed with the help of eqs.(2),(3) and the equalities

$$0 = \langle f | [V_G^{(-)}, G^{(-)}] | i \rangle = \sum_S \left(\langle f | V_G^{(-)} | S \rangle \langle S | G^{(-)} | i \rangle - \langle f | G^{(-)} | S \rangle \langle S | V_G^{(-)} | i \rangle \right). \quad (5)$$

The commutator vanishes because of the fact that $(\tau^{(-)})^2 = 0$. As a result of the transformations performed with the help of eqs.(2)-(5) we get the final expression for the nuclear $2\nu\beta\beta$ - decay amplitude:

$$M_G = \omega_G^{-2} \sum_S \langle f | V_G^{(-)} | S \rangle \langle S | V_G^{(-)} | i \rangle \omega_S^{-1}. \quad (6)$$

Summation in this equation formally includes the state $|G\rangle$ as well, because $\langle G | V_G^{(-)} | i \rangle = 0$.

Eqs. (1) and (6) are equivalent. However, the $2\nu\beta\beta$ -decay hindrance due to the presence of the collective states (IAS and GTS) can be explicitly taken into account in eq.(6) provided that the state $|G\rangle$ is, respectively, taken as one of these states. Indeed, if the above states each exhaust 100% of the corresponding particle-hole strength, i.e. $|G\rangle \sim G^{(-)}|i\rangle$, then matrix elements $\langle S | V_G^{(-)} | i \rangle$ and, therefore, the amplitudes M_G are equal to zero. It is now clear, in a model-independent way, that if the collective state $|G\rangle$ exhausts most of the sum-rule strength, the $2\nu\beta\beta$ -decay will be strongly suppressed.

For the above reasons the use of eq.(6) allows one to bypass the difficulty connected with the use of current versions of QRPA which are based on eq.(1). To show it we first consider nuclei without nucleon pairing using the phenomenological nuclear mean field U and the isovector part of the Landau-Migdal particle-hole interaction F . The mean field contains isoscalar, isovector, spin-orbit and Coulomb parts:

$$U = U_0(r) + U_\tau(r)\tau^{(3)} + U_{\sigma l}(r)(\vec{\sigma}\vec{l}) + U_C(r)(1 - \tau^{(3)})/2. \quad (7)$$

The interaction

$$F = (F_\tau + F_{\sigma\tau}\vec{\sigma}_1\vec{\sigma}_2)\vec{\tau}_1\vec{\tau}_2\delta(\vec{r}_1 - \vec{r}_2) \quad (8)$$

contains two phenomenological parameters $F_g (g = \tau, \sigma\tau)$ which are widely used within in the theory of finite Fermi-systems [15]. Let $\rho(r) = (\rho^{(+)}(r) + \tau^{(3)}\rho^{(-)}(r))/2$ be nucleon density: $\rho^{(\pm)}(r) = \rho^n(r) \pm \rho^p(r)$, ρ^n and ρ^p are the neutron and proton densities, respectively. The following conditions

$$U_\tau(r) = F_\tau\rho^{(-)}; \quad U_C(r) = e^2 \int \rho^p(r_1) |\vec{r} - \vec{r}_1|^{-1} d\vec{r}_1 \quad (9)$$

allow one to calculate two terms in eq.(7) in a self-consistent way (the first condition is discussed in e.g. ref.[16]).

Let us turn to the operator $V_G^{(-)} = \sum_a V_g^{(-)}(a)$ (2). It can be shown using the coordinate representation of the RPA equations that:

$$\begin{aligned} -i\hbar\dot{g}^{(-)} &= [h, g^{(-)}] - 2F_g[\rho, g^{(-)}] , \\ V_g^{(-)} &= [h, g^{(-)}] - 2F_g[\rho, g^{(-)}] - \Delta_G g^{(-)}. \end{aligned} \quad (10)$$

Here, $h = t + U$ is the single-particle Hamiltonian. Assuming $|G\rangle \sim G^{(-)}|i\rangle$, we find an approximate expression for the energy Δ_G using the equality $\langle G|V_G^{(-)}|i\rangle = 0$ and eq.(2):

$$\Delta_G \cong -i\hbar \frac{\langle i|[G^{(+)}, \dot{G}^{(-)}]|i\rangle}{\langle i|[G^{(+)}, G^{(-)}]|i\rangle} . \quad (11)$$

This expression can also be obtained by means of the energy weighted sum rule where Δ_G equals to the mean energy of Fermi or GT excitations, respectively. This formula has acceptable accuracy because the IAS and GTS exhaust most of the corresponding particle-hole strength.

For the case of Fermi transitions ($g^{(\pm)} = \tau^{(\pm)}$) we obtain from eqs.(7)-(11):

$$V_\tau^{(-)} = (U_C(r) - \Delta_C)\tau^{(-)}; \quad \Delta_C = (N - Z + 2)^{-1} \int U_C(r)\rho^{(-)}(r) d\vec{r}. \quad (12)$$

These well-known equations are a result of the approximate conservation of isospin symmetry in medium and heavy nuclei, where the mean Coulomb field is the main source of violation of this symmetry. Similar calculations for the case of Gamow-Teller transitions ($g^{(\pm)} = \vec{\sigma}\tau^{(\pm)}$) lead to the following expression:

$$\begin{aligned} V_{\sigma\tau}^{(-)} &= (U_C(r) - \Delta_C)\vec{\sigma}\tau^{(-)} + (U_{\sigma l}(r)[\vec{\sigma}\vec{l}, \vec{\sigma}] - \Delta_{\sigma l}\vec{\sigma})\tau^{(-)} + \\ &\quad (2\delta F\rho^{(-)}(r) - \Delta_{\delta F})\vec{\sigma}\tau^{(-)}, \end{aligned} \quad (13)$$

where Δ_C is determined by eqs.(12), $\delta F = F_\tau - F_{\sigma\tau}$,

$$\begin{aligned} \Delta_{\sigma l} &= -\frac{2}{3}(N - Z + 2)^{-1} \langle i | \sum_a U_{\sigma l}(r_a) \vec{\sigma}_a \vec{l}_a | i \rangle, \\ \Delta_{\delta F} &= 2(N - Z + 2)^{-1} \delta F \int (\rho^{(-)}(r))^2 d\vec{r} \end{aligned} \quad (14)$$

and

$$\Delta_{\sigma\tau} = \Delta_C + \Delta_{\sigma l} + \Delta_{\delta F} . \quad (15)$$

The three terms in eq.(13) arise as a result of spin-isospin symmetry violation (this symmetry may be called a "projection on the charge-exchange channel" of the spin-isospin SU(4) - symmetry [17]). However, due to the smooth radial dependence of both the mean Coulomb field and the excess neutron density the spin-orbit part of the nuclear mean field is the main source of the redistribution of the Gamow-Teller strength from the GTS to its satellites. In other words, only the second term in eq.(2) needs to be taken into consideration in the analysis of the Gamow-Teller $2\nu\beta\beta$ - decay amplitude by means of eq.(6). The other terms are only important for the calculation of the energy of the GTS (15).

The equations equivalent to eqs. (10) have not yet been formulated for nuclei with pairing. Nevertheless, one can state that the operator $V_{\tau}^{(-)}$ (12) is not changed since pairing forces conserve isospin symmetry. In the case of Gamow-Teller transitions the operator $V_{\sigma\tau}^{(-)}$ (13) is expected to be somewhat modified due to that part of the particle-particle interaction, which violates the "projection" of SU(4) - symmetry. The relative contribution of this interaction to $V_{\sigma\tau}^{(-)}$ can be roughly estimated as the ratio of the energy gap to the mean single-particle spin-orbit energy-splitting, which is rather small (no more than 20%). This contribution is omitted in the following analysis.

In view of a higher degree of the isospin-symmetry conservation as compared with the spin-isospin-symmetry conservation one can conclude that $M_{GT} \gg M_F$. For this reason we consider in the following only the Gamow-Teller $2\nu\beta\beta$ - decay amplitude.

3. Evaluation of the decay amplitude within a simplest version of the approach

As a first step in evaluating the amplitude M_{GT} with the help of the eq.(6) we use the simplest approximation for describing the corresponding states in isobaric nuclei. Namely, we use the BCS model for describing subsystems with strong nucleon pairing and the pair-vibration model for describing "magic \pm two nucleons" subsystems.

Within this approximation the nucleon densities $\rho^{(\pm)}(r)$ are equal to

$$\rho^{(\pm)}(r) = \frac{1}{4\pi} \left(\sum_{\nu} R_{\nu}^2(r)(2j_{\nu} + 1)n_{\nu} \pm \sum_{\pi} R_{\pi}^2(r)(2j_{\pi} + 1)n_{\pi} \right), \quad (16)$$

where $R_\lambda(r)$ are the radial single-neutron ($\lambda = \nu$) and single-proton ($\lambda = \pi$) wave functions; $\lambda = n_r, l, j$ is the set of the single-particle quantum numbers, and n_λ are occupation numbers satisfying the equations:

$$\sum_{\nu} (2j_{\nu} + 1)n_{\nu} = N + 2, \quad \sum_{\pi} (2j_{\pi} + 1)n_{\pi} = Z. \quad (17)$$

For nuclei with strong pairing in any nucleon subsystem the occupation numbers are $n_\lambda = v_\lambda^2 = 1 - u_\lambda^2$, where v_λ and u_λ are the Bogoliubov-transformation coefficients. In the case that a nucleon subsystem is the "magic \pm two nucleons" subsystem the occupation factors are: $n_\lambda = n_\lambda^m + c_\lambda^2(1 - 2n_\lambda^m)$, where n_λ^m are the occupations numbers for the magic subsystem; c_λ are the coefficients, which determine the pair-vibration wave function and satisfy the normalization condition: $\sum_\lambda c_\lambda^2(1 - 2n_\lambda^m)(2j_\lambda + 1) = \pm 2$.

The terms Δ_C , $\Delta_{\sigma l}$ and $\Delta_{\delta F}$ determining the GTS energy (15) can also be calculated easily according to eqs.(12),(14). In particular, we have

$$\Delta_{\sigma l} = -\frac{8}{3}(N - Z + 2)^{-1} \sum_{\lambda=\pi,\nu} n_\lambda \langle \lambda | U_{\sigma l}(r) | \lambda \rangle (j_\lambda - l_\lambda) l_\lambda (l_\lambda + 1), \quad (18)$$

where $\langle \pi | U_{\sigma l}(r) | \nu \rangle = \int R_\pi(r) R_\nu(r) U_{\sigma l}(r) r^2 dr$ and n_λ are occupation numbers satisfying eqs.(17).

Calculation of the amplitude M_{GT} according to eq.(6) within the framework of the BCS model results in the expression:

$$M_{GT} = e_q^2 \omega_{GTS}^{-2} \sum_{\pi,\nu} \left((2l_\pi + 1)(j_\pi - j_\nu) \langle \pi | U_{SO}(r) | \nu \rangle - \Delta_{SO} \langle \pi | \nu \rangle \right)^2 \times \langle \pi || \sigma || \nu \rangle^2 u_\pi v_\pi u_\nu v_\nu \omega_{\pi\nu}^{-1}. \quad (19)$$

Here, $\langle \pi || \sigma || \nu \rangle$ is the reduced matrix element; $\omega_{\pi\nu} = \mathcal{E}_\pi + \mathcal{E}_\nu$ is the excitation energy of the two-quasiparticle state, $\mathcal{E}_\lambda = \sqrt{(\epsilon_\lambda - \mu)^2 + \Delta^2}$ is the single-quasiparticle energy for the subsystem with strong nucleon pairing, ϵ_λ is the energy of single-particle level, and Δ is the energy gap. If the nucleon subsystem is magic in final (initial) state and "magic \pm two nucleon" in initial (final) state, we use c_λ instead of $u_\lambda v_\lambda$ in eq. (19) and $\mathcal{E}_\lambda = |\Delta + \epsilon_\lambda - \epsilon_1|$ for particle pair-vibrations or $\mathcal{E}_\lambda = |\Delta + \epsilon_0 - \epsilon_\lambda|$ for hole pair-vibrations, where ϵ_1 (ϵ_0) is the energy of the first empty (last filled) single-particle level in the magic subsystem. Here, 2Δ is the pair-vibration state energy, which coincides with the pairing energy P for the subsystem "magic \pm two nucleons". The hindrance factor h_{GT} can be estimated as follows: $h_{GT} = M_{GT}/M_{GT}^0$, where

amplitude

$$M_{GT}^0 = e_q^2 \sum_{\pi, \nu} \langle \pi | \nu \rangle^2 \langle \pi | \sigma | \nu \rangle^2 u_\pi v_\pi u_\nu v_\nu \omega_{\pi\nu}^{-1} . \quad (20)$$

is evaluated according to the initial equation (1) with the use of the BCS (or BCS and pair-vibration) model without consideration of the hindrance caused by the presense of the collective state.

4. Results of the calculations and summary

The parameterization of the isoscalar part of the phenomenological nuclear mean field including the spin-orbit term is given e.g. in ref. [18]. The strength $F_\tau = 300 \text{ MeV fm}^3$ in eq. (8) is chosen to describe the experimental neutron and proton binding-energy difference for nuclei with rather large neutron excess $^{48}\text{Ca}, ^{68}\text{Ni}, ^{132}\text{Sn}, ^{208}\text{Pb}$ [19], for which the difference is mainly determined by the symmetry potential and mean Coulomb field. The strength $F_{\sigma\tau} = 255 \text{ MeV fm}^3$ in eq.(8) is chosen following the suggestion of ref. [15]. The parameters $\mu_{n,p}$ and $\Delta_{n,p}$, obeying the constraints of eqs. (17), were found for each subsystem with strong nucleon pairing to describe the experimental pairing energies $P = 2 \min_\lambda \mathcal{E}_\lambda$ taken according to ref. [19]. The pair-vibration state energies $2\Delta = P$ are calculated using experimental pairing energies [19]. Coefficients c_λ determining the pair-vibration state wave function for a nucleon subsystem "magic + or - two nucleons", are calculated using $c_\lambda \sim (1 - 2n_\lambda^m)/(\epsilon_\lambda - \epsilon_1 + 2\Delta)$ or $c_\lambda \sim (1 - 2n_\lambda^m)/(\epsilon_0 - \epsilon_\lambda + 2\Delta)$ with taking into account the normalization conditions given above [20]. For all considered nuclei the single-particle basis including all bound states as well as the quasi-bound states up to $\sim 5 \text{ MeV}$ is used. If the nucleon subsystem in a parent nucleus is "magic \pm two nucleons", and in the product-nucleus is "magic \pm four nucleons", the calculations were performed within the framework of the BCS model with the use of u_λ, v_λ factors calculated for a "magic \pm four nucleons" subsystem.

Results of the calculations and corresponding experimental data are given in Table 1. The M_{GT} amplitudes calculated using eq.(19) are given in column 5. The M_{GT}^0 amplitudes were calculated from eq. (20) with the use of the same model parameters and the same number of basis states. The calculated hindrance factors h_{GT} are given in column 6. Half-lives $T_{1/2}^{calc}$ calculated by formula $(T_{1/2})^{-1} = G_{2\nu} |M_{GT}|^2$ are given in column 8. The lepton factors $G_{2\nu}$ taken from refs. [2,13,14] are also given (column 3). We calculated also the $2\nu\beta^+\beta^+$ half-lives (including the electron capture) for those nuclei, for which relevant experimental data is expected to be forthcoming.

In this work the initial formula for the nuclear $2\nu\beta\beta$ - decay amplitude is transformed in a model-independent way to an expression, which takes explicitly into account the decay-rate hindrance due to the presence of the collective GTS. To apply the transformed formula in its simplest version we use: (i) the fact that the GTS exhausts the most of the Gamow-Teller strength; (ii) the BCS and the pair-vibration models for describing the corresponding states of isobaric nuclei. The formula for the amplitude M_{GT} , obtained with the use of the above approximations, is the sum of the positive-sign terms and, therefore, is stable to reasonable variations of model parameters. Calculations are performed for a wide range of nuclei with the use of the parameters found from independent data. Except for Te isotopes the calculated amplitudes M_{GT} within the factor 2–3 are in agreement with the corresponding amplitudes M_{GT}^{exp} deduced from experimental data.

The present approach for evaluating the nuclear $2\nu\beta\beta$ -decay amplitude is incorporating in a model-independent way the mechanism for the rather strong suppression of the decay-rate and is thus to be preferred over conventional approaches. The use of the present approach in QRPA requires further investigation.

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Table 1. Calculated M_{GT} and $T_{1/2}$ values in comparison with the corresponding experimental data. ^{150}Nd is considered as a spherical nucleus. The lepton factors $G_{2\nu}$ and calculated hindrance factors are also given. m_e is the electron mass.

parent nucleus	type of decay	$G_{2\nu}$ $years^{-1} m_e^2$	$M_{GT}^{exp.}$ m_e^{-1}	M_{GT} m_e^{-1}	h_{GT}	$T_{1/2}^{exp.}$ $years$	$T_{1/2}^{calc.}$ $years$
^{76}Ge	$\beta^-\beta^-$	1.317×10^{-19} [13]	0.0919	0.0387	0.131	0.9×10^{21} [21]	5.0×10^{21}
			0.0737			1.43×10^{21} [22]	
^{78}Kr	$ec\ ec$	1.957×10^{-21} [14]		0.0285	0.090		6.2×10^{23}
	β^+ec	1.174×10^{-21} [14]					1.0×10^{24}
^{82}Se	$\beta^-\beta^-$	4.393×10^{-18} [13]	0.0459	0.0295	0.093	1.08×10^{20} [23]	2.6×10^{20}
^{96}Zr	$\beta^-\beta^-$	1.953×10^{-17}	0.0362	0.0678	0.324	3.9×10^{19}	1.1×10^{19}
^{96}Ru	$ec\ ec$	6.936×10^{-21} [14]		0.1005	0.338		1.4×10^{22}
	β^+ec	1.148×10^{-21} [14]				$> 6.7 \times 10^{16}$ [24]	8.6×10^{22}
^{100}Mo	$\beta^-\beta^-$	9.553×10^{-18} [13]	0.0954	0.1606	0.329	1.15×10^{19} [25]	4.1×10^{18}
^{106}Cd	$ec\ ec$	1.573×10^{-20} [14]		0.1947	0.319		1.7×10^{21}
	β^+ec	1.970×10^{-21} [14]				$> 6.6 \times 10^{18}$ [26]	1.3×10^{22}
^{116}Cd	$\beta^-\beta^-$	8.000×10^{-18} [2]	0.0754	0.0788	0.258	2.25×10^{19} [2]	1.2×10^{19}
^{124}Xe	$ec\ ec$	5.101×10^{-20} [14]		0.0528	0.085		7.0×10^{21}
	β^+ec	4.353×10^{-21} [14]					8.2×10^{22}
^{128}Te	$\beta^-\beta^-$	8.624×10^{-22} [13]	0.0123	0.0529	0.090	7.7×10^{24} [27]	4.1×10^{23}
^{130}Te	$\beta^-\beta^-$	4.849×10^{-18} [13]	0.0087	0.0468	0.085	2.7×10^{21} [27]	9.4×10^{19}
^{130}Ba	$ec\ ec$	4.134×10^{-20} [14]		0.0568	0.082	$> 4 \times 10^{21}$ [28]	7.5×10^{21}
	β^+ec	1.387×10^{-21} [14]				$> 4 \times 10^{21}$ [28]	2.2×10^{23}
^{136}Xe	$\beta^-\beta^-$	4.870×10^{-18} [13]	0.0299	0.0341	0.088	$> 2.3 \times 10^{20}$ [29]	1.1×10^{20}
^{136}Ce	$ec\ ec$	3.988×10^{-20} [14]		0.0512	0.081		9.6×10^{21}
	β^+ec	6.399×10^{-22} [14]					6.0×10^{23}
^{150}Nd	$\beta^-\beta^-$	1.200×10^{-16} [13]	0.0221	0.0642	0.182	1.7×10^{19} [30]	2.0×10^{18}
			0.0304			9×10^{18} [31]	

The table should be put in the end of Sect. 4